

Dynamical stabilization and time in open quantum systems

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Abstract

The meaning of time in an open quantum system is considered under the assumption that both, system and environment, are quantum mechanical objects. The Hamilton operator of the system is non-Hermitian. Its imaginary part is the time operator. As a rule, time and energy vary continuously when controlled by a parameter. At high level density, where many states avoid crossing, a dynamical phase transition takes place in the system under the influence of the environment. It causes a dynamical stabilization of the system what can be seen in many different experimental data. Due to this effect, time is bounded from below: the decay widths (inverse proportional to the lifetimes of the states) do not increase limitless. The dynamical stabilization is an irreversible process.

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I. INTRODUCTION

The problem of time is considered in very many papers starting from the very beginning of quantum mechanics. For example, the derivation of the uncertainty relation between space and momentum can be found in every textbook on quantum mechanics. However, the uncertainty relation between energy and time could not be derived convincingly up to now. According to Pauli [1], the reason is that energy is bounded from below while this is not the case for time and, furthermore, time varies continuously in contrast to energy. A critical consideration of the discussions of the time energy uncertainty relation is given in [2]. The authors point out that energy E characterizes the quantum system while time t is measured by an external clock.

This point of view is followed up in recent studies [3]: *it is important in the application of a given time energy uncertainty relation to state precisely what kind of measurement is being made and to specify accordingly the meaning of the Δt involved, e.g. does it refer to the accuracy of measurement, to the duration of measurement or perhaps to the lifetime of a decaying state.* Also the derivation of the time dependent Schrödinger equation is considered [4]: *This derivation, unlike those presented in quantum mechanics textbooks but in the spirit of Schrödinger's original approach to the problem, acknowledges that time enters quantum mechanics only when an external force on the quantum system is considered classically.* Starting from a fully time-independent formulation of quantum mechanics, it is possible systematically to derive the time-dependent Schrödinger equation for a quantum system in the approximation that the environment is treated semiclassically. The derivation relies on the assumption of a closed object comprising system plus environment [4].

In all these considerations the Hamiltonian of the system is assumed to be Hermitian, and the lifetime of a decaying state is calculated perturbatively. The situation is another one when both, system and environment, are quantum mechanical objects, i.e. the environment is the continuum of scattering wavefunctions into which the system is embedded. In such a case, the Hamilton operator of the system is non-Hermitian and the lifetime of a decaying state is calculated non-perturbatively [5].

It is the aim of the present paper to consider the meaning of time in an open quantum system described by a non-Hermitian operator the eigenvalues of which are complex. The basic equations are taken from the review [5]. It turns out that a natural definition of time

in an open quantum system is related to the decay widths of the states and, furthermore, to the dynamical stabilization occurring in the system under the influence of the environment. Time, defined in such a manner, is bounded from below.

The paper is organized in the following manner. In section II it is shown why in an open quantum system with many levels the non-Hermitian operator H_{eff} appears and how it looks like. Using this model, the properties of discrete and narrow resonance states are sketched in section III. In both cases, exceptional points play an important role. Since there is much confusion in the literature on these singular points, their properties are sketched in the appendixes A to E from a unified point of view and compared to the results of experimental studies. The relation of the imaginary part of the eigenvalues of H_{eff} to time is considered in section IV while in the following section V the decay rate is calculated. The phenomenon of a dynamical phase transition in an open quantum system is explained in section VI. In section VII, a few examples of dynamical phase transitions, that are found experimentally, are given. In any case, they cause a dynamical stabilization of the system. Based on these results, it is possible to define *time* in an open quantum system and to discuss its properties (section VIII). The results are summarized in the last section IX.

II. NON-HERMITIAN HAMILTON OPERATOR OF AN OPEN QUANTUM SYSTEM

The definition of an *open quantum system* used in the present paper is the following: a quantum system is considered to be open when it is embedded into the environment of the continuum of scattering wavefunctions. According to this definition, the system is localized while the environment is extended infinitely. This type of embedding of a quantum system into an environment *always* exists. The environment may be changed by external forces, e.g. by a laser in the case of atoms (examples are shown in [6]). It can however *not* be deleted completely. When allowed by the energy of the system (and no special selection rules hold), most states of the system decay into the continuum of scattering wavefunctions and have a finite lifetime. Otherwise they are discrete.

A method being very suitable for the description of this situation, is a projection operator formalism with projection onto system and environment, respectively. In the following, some basic equations of such a formalism are given, for details see the review [5]. First the

Schrödinger equations in the two subspaces (with the Hamiltonian H_B and H_c , respectively) have to be solved and the projection operators Q and P have to be defined,

$$(H_B - E_k^B) \Phi_k^B = 0 \quad \rightarrow \quad Q = \sum_{k=1}^N |\Phi_k^B\rangle \langle \Phi_k^B| \quad (1)$$

$$(H_c - E) \xi_c^E = 0 \quad \rightarrow \quad P = \sum_{c=1}^C \int_{-\infty}^{\infty} dE |\xi_c^E\rangle \langle \xi_c^E|. \quad (2)$$

The Φ_k^B are the wavefunctions describing the N discrete states of the closed many-particle system, while the ξ_c^E are the scattering wavefunctions of the environment consisting of C continua (ξ_c^E is written instead of $\xi_c^{E(+)}$ for convenience). The Schrödinger equation in the whole function space with discrete and scattering states reads

$$(H^{\text{full}} - E) \Psi_c^E = 0 \quad (3)$$

with the Hermitian operator $H^{\text{full}} = H_{QQ} + H_{QP} + H_{PQ} + H_{PP}$ and $Q + P = 1$ where $H_{QQ} \equiv QH_Q$ and so on. The coupling matrix elements between system and environment are

$$\gamma_{kc}^0 = \sqrt{2\pi} \langle \Phi_k^B | H_{QP} | \xi_c^E \rangle. \quad (4)$$

The solution of the full problem (3) is

$$\Psi_c^E = \xi_c^E + \sum_{k=1}^N \Omega_k \cdot \frac{\langle \Phi_k^* | H_{QP} | \xi_c^E \rangle}{E - z_k} \quad (5)$$

with the non-Hermitian operator

$$H_{\text{eff}} = H_{QQ} + H_{QP} G_P^{(+)} H_{PQ} \equiv H_B + V_{BC} G_C^{(+)} V_{CB} \quad (6)$$

and

$$(H_{\text{eff}} - z_k) \Phi_k = 0; \quad z_k \equiv E_k - \frac{i}{2} \Gamma_k \quad (7)$$

after diagonalization. Here $E_k = E_k(E)$ and $\Gamma_k = \Gamma_k(E)$ are, respectively, the position in energy and the decay width (inverse lifetime) of the state k at the energy E , see equations (3) and (5).

Position and width of the resonance state k are energy independent numbers $E_k = E_k^p$ and $\Gamma_k = \Gamma_k^p$ only when the state k is not overlapped by another resonance state and,

furthermore, it is far from a decay threshold. These numbers can be obtained also, as usually, from the poles of the S matrix, see [7]. The functions $z_k = z_k(E)$ describe, in any case, the line shape of resonances correctly. Further, $G_P^{(+)} = P(E - H_{PP})^{-1}P$ is the Green function in the P subspace and $\Omega_k = (1 + G_P^{(+)} H_{PQ}) \Phi_k$ is the wavefunction of the resonance state. The eigenfunctions of H_{eff} are biorthogonal, $\langle \Phi_k^* | \Phi_l \rangle = \delta_{k,l}$ and $\langle \Phi_k | \Phi_k \rangle \equiv A_k \geq 1$.

The Hamiltonian H_{eff} consists *formally* of a first-order and a second-order interaction term. The second-order term via the continuum determines the dynamics of the system at high level density. It leads to the principal value integral

$$\text{Re} \langle \Phi_i^B | H_{\text{eff}} | \Phi_j^B \rangle - E_i^B \delta_{ij} = \frac{1}{2\pi} \sum_{c=1}^C \mathcal{P} \int_{\epsilon_c}^{\epsilon'_c} dE' \frac{\gamma_{ic}^0 \gamma_{jc}^0}{E - E'} \quad (8)$$

and the residuum

$$\text{Im} \langle \Phi_i^B | H_{\text{eff}} | \Phi_j^B \rangle = -\frac{1}{2} \sum_{c=1}^C \gamma_{ic}^0 \gamma_{jc}^0. \quad (9)$$

When $i = j$, the expression $\text{Re} \langle \Phi_i^B | H_{\text{eff}} | \Phi_j^B \rangle$ gives the shift in energy of the state i due to the interaction between the state and the large system it is part of, i.e. due to the interaction of the state i with the environment (subspace P). This is the *self-energy* of the state which is analog to the Lamb shift known in atomic physics. When $i \neq j$, (8) describes the energy shift of the state i due to its coupling to another state $j \neq i$ via the environment (continuum of scattering wavefunctions). These couplings to the different states j cause, at high level density, collectively contributions to the energy shift of the state i . In atomic physics, contributions of such a type are studied newly experimentally and called collective Lamb shift [8, 9].

The energy window coupled directly to the continuum is $\epsilon_c \leq E \leq \epsilon'_c$. In nuclei $\epsilon'_c \rightarrow \infty$ while ϵ_c denotes the lowest threshold for emission of a particle. The energy shift $\Delta_k = E_k - E_k^B$, including the corresponding corrections arising from the coupling of different states via the continuum, can *not* be simulated by two-body forces [10]. It is a global property, see [11], and is *not* contained in any standard calculation with a Hermitian Hamilton operator for the many-body problem.

The method sketched here for the description of an open quantum system is not the only one. The advantage of the explicit consideration of H_{eff} in the model described above, consist, above all, in the fact that *the many-body problem in the Q subspace has to be solved only once*, since it is energy independent, see (1). At low level density, $H_{\text{eff}} \approx H_B \equiv H_{QQ}$,

the second-order term can be treated perturbatively, i.e. the Hamiltonian H_{eff} is almost Hermitian, corresponding to the assumption of standard quantum physics. At high level density, however, the second-order term can *not* be treated perturbatively. It induces a global mixing of the states according to (8) [11], and deviations from standard quantum physics will occur. They appear most clearly when the number of decay channels (continua) is small, especially for $C = 1$.

III. DISCRETE AND NARROW RESONANCE STATES

The eigenvalues of the effective Hamiltonian H_{eff} , equation (6), may be real or complex. In the first case, the eigenstates are discrete states while they are resonance states in the second case. The boundary conditions are different for the two different types of states. In both cases, the trajectories of the eigenvalues traced as a function of any control parameter avoid crossing, usually. The corresponding crossing points (called mostly *exceptional points*) of two eigenvalue trajectories can be found by analytical continuation. They play an important role for the dynamics of open quantum systems. The eigenvalues and eigenfunctions at and in the neighborhood of an exceptional point (including the experimental proof of their main features) are given in appendix A and B, respectively.

Resonance states are coupled directly to the continuum, $\epsilon_c \leq E_k \leq \epsilon'_c$. The eigenvalues z_k of H_{eff} are complex, generally. According to (5), the scattering wavefunction $\Psi_{c \text{ int}}^E$ inside the system can be represented in the set $\{\Phi_k\}$ of the biorthogonal eigenfunctions of H_{eff} , i.e.

$$\Psi_{c \text{ int}}^E = \sum_{k=1}^N \frac{\langle \Phi_k^* | H_{QP} | \zeta_c^E \rangle}{E - z_k} \Phi_k. \quad (10)$$

The expression $\langle \Phi_k^* | \Phi_l \rangle$ is a complex number, and a consistent normalization of the Φ_k requires $\text{Im} \langle \Phi_k^* | \Phi_k \rangle = 0$. This corresponds to some rotation such that the phases of the eigenfunctions relative to one another are not rigid (when traced as a function of a certain parameter). Instead, it holds $0 \leq \rho \leq 1$ (for details see [5]) for the phase rigidity

$$\rho = e^{2i\theta} \frac{\int dr ([\text{Re} \Psi_{c \text{ int}}^E]^2 - [\text{Im} \Psi_{c \text{ int}}^E]^2)}{\int dr ([\text{Re} \Psi_{c \text{ int}}^E]^2 + [\text{Im} \Psi_{c \text{ int}}^E]^2)} \quad (11)$$

where θ is the rotation angle (see appendix C for the definition of the phase rigidity r_k in the case of a two-level system). Only at low level density and far from avoided level crossings, $\langle \Phi_k^* | \Phi_l \rangle \approx \langle \Phi_k | \Phi_l \rangle$ and $\rho \approx 1$ such that standard Hermitian quantum physics is

a good approximation. Otherwise, spectroscopic redistribution processes take place, $\rho < 1$, the Schrödinger equation has a nonlinear source term (see appendix D) and time reversal symmetry is broken (see appendix E). Here, the description of the system by standard Hermitian quantum mechanics breaks down.

The breakdown of standard quantum mechanics can be understood in the following manner. When $1 > \rho > 0$ the states with the wavefunctions $\Psi_{c \text{ int}}^E$ avoid crossing (when controlled by a parameter) and become mixed globally such that a few states of the system can align *hierarchically* (i.e. step by step [5]) with the scattering states ξ_c^E of the environment. As a result, their decay widths Γ_k become large. Full alignment is reached for $\rho = 0$. The alignment of a few states $\Psi_{c \text{ int}}^E$ with the channel wavefunction ξ_c^E occurs by trapping other resonance states, i.e. by (partial or complete) decoupling them from the environment. This is nothing but width bifurcation: the widths of a few states become large while the widths of the other ones become small by varying the control parameter. This scenario occurs in the vicinity of exceptional points [5, 12]. When $\rho < 1$, the system can not be described perturbatively, and standard Hermitian quantum physics fails (compare Appendixes C to E for the two-level system). Similar results are obtained by using other methods, e.g. [13–16].

The influence of the continuum of scattering wavefunctions (P subspace) onto the *discrete states* of the system with energies E_k beyond the energy window $\epsilon_c \leq E \leq \epsilon'_c$, seems to be much less important. For discrete states H_{eff} is non-Hermitian, but the eigenvalues $z_k = E_k$ are real, the eigenfunctions Φ_k are orthogonal, $\langle \Phi_i | \Phi_k \rangle = \delta_{ik}$, and the phase rigidity is $\rho = 1$. However, $E_k \neq E_k^B$. According to (8), the energy shift $\Delta_k = E_k - E_k^B \neq 0$ is caused by the coupling of the state k to the environment, i.e. by the embedding of the system into the continuum of scattering wavefunctions. By this, many-body forces are induced in the system [10]. Discrete states avoid crossing and, at a critical value of the control parameter, the two states are exchanged as known for about 80 years [17]. The only difference to the avoided crossing of resonance states is that discrete states never cross. The corresponding crossing point can be found only by analytical continuation into the continuum [18].

When the level density is high, many discrete levels avoid crossing (when traced as a function of a control parameter). For illustration let us consider an A particle system. In this case, the induced many-body forces cause a global mixing of the discrete states in a finite parameter range, see [11]. Finally, an *aligned* discrete state is formed with the structure *bound particle + (A-1) particle residual system* which corresponds to the structure of the

decay channel *unbound particle + (A-1) particle residual system* (the quantum numbers of particle and residual system are the same in both cases and (A-1) denotes the number of particles of the localized residual system after emission of one particle into the continuum). This discrete state is the analog to an aligned resonance state above particle decay threshold. The only difference between these two states is that the energy of the preformed aligned discrete state is too small and does not allow the emission of one particle from the system, while the decay width of the aligned resonance state is large, corresponding to a short lifetime of this state.

It should be mentioned that energy conservation is not the only source for an eigenvalue of H_{eff} to be real. $\text{Im}(z_k) \equiv -\Gamma_k/2 = 0$ is possible also due to selection rules (according to the corresponding quantum numbers of the states), or because of width bifurcation appearing at high level density (causing the so-called bound states in the continuum [5]). Another source for real eigenvalues of a non-Hermitian operator appears in PT symmetric systems (where P and T denote parity and time, respectively) [19] and can be observed in optics due to the formal equivalence of the optical wave equation in PT symmetric optical lattices to the quantum mechanical Schrödinger equation [20]. The relation of these results to the properties of open quantum systems as discussed above, is considered in [12], see also appendix A.

IV. THE IMAGINARY PART OF THE EIGENVALUES OF THE OPERATOR H_{eff}

In order to study the physical meaning of $\text{Im}(z_k) = -\Gamma_k/2$, equation (7), the behavior of three neighboring resonances in a two-dimensional quantum billiard connected to a single waveguide is investigated theoretically in [21]. A measurable quantity derived from the reflection coefficient $R(E)$ is the *Wigner Smith time delay function*

$$\tau_w = \frac{d\Theta}{dE} . \quad (12)$$

It is the time the wave spends inside the billiard. The energies E_k^p and widths Γ_k^p of the resonance states can be found from the *poles of the function* $R(E)$ analytically continued into the lower complex plane.

In [21], contour and surface plot of $\ln(\tau_w)$ and the motion of the corresponding three resonance poles by varying the coupling between the waveguide and the resonator are cal-

culated. At weak coupling to the waveguide, the three resonances are seen clearly. As the coupling to the waveguide increases, the lifetimes of all three states decrease, as expected. As the resonances start to influence one another, the states attract each other in energy, two of them become trapped while the third one becomes short-lived. At further increasing coupling, the lifetimes of the two trapped resonance states increase, contrary to expectation. The lifetime of the short-lived state is, at large opening, so short that it practically disappears when plotting the time delay. The motion of the poles is reflected in the time delay function. The calculations have shown further that the interference between the resonance states leads to a mixing of their wavefunctions with respect to the eigenfunctions of the closed resonator (defined by decoupling the resonator from the waveguide).

Some years ago, the dynamics of resonance states is studied experimentally by means of a flat microwave resonator connected to a waveguide where the coupling strength between resonator and waveguide can be varied by hand [22]. In this experiment, the microwaves enter the billiard through a slit, the opening of which can be varied. The motion of the resonance poles as a function of the opening of the slit is traced starting close to the real axis and following them into the region of overlapping resonances. The results verify the resonance trapping effect discussed above. This experimental proof does not depend on any model assumptions. Meanwhile, the resonance trapping effect has been investigated and verified in many other theoretical and experimental studies performed on different systems (see section VII and review [5]).

As a result of these studies, the physical meaning of the imaginary part $\text{Im}(z_k)$ of the eigenvalues of the non-Hermitian Hamilton operator H_{eff} is directly related to the time τ_w . This relation holds also at high level density where the system as a whole is dynamically stabilized (see section VI).

V. DECAY RATE AT HIGH LEVEL DENSITY

The time dependent Schrödinger equation for the scattering wavefunctions $\Psi_{c \text{ int}}^E$ inside the system reads

$$H^{\text{full}} \Psi_{c \text{ int}}^E(t) = i\hbar \frac{\partial}{\partial t} \Psi_{c \text{ int}}^E(t) \quad (13)$$

with H^{full} defined in (3) and $\Psi_{c \text{ int}}^E$ in (10). The right solutions $|\Psi_{c \text{ int}}^E\rangle$ may be represented by an ensemble of resonance states k that describes the decay of the localized part of the system at the energy E ,

$$|\Psi_{c \text{ int}}^E(t)\rangle = e^{-iH_{\text{eff}} t/\hbar} |\Psi_{c \text{ int}}^E(t_0)\rangle = \sum_{k=1}^N e^{-iz_k t/\hbar} c_{ck}^E |\Phi_k\rangle \quad (14)$$

with $c_{ck}^E = \langle \Phi_k^* | V | \xi_c^E \rangle / (E - z_k)$ according to (10). By means of (14) and the corresponding expression for the left solution of (13), the population probability $\langle \tilde{\Psi}_c(t) | \tilde{\Psi}_c(t) \rangle = \sum_{\lambda} c_{c\lambda}^2 e^{-\Gamma_{\lambda} t/\hbar}$ with the energy averaged values c_{ck} can be defined. The decay rate reads [23]

$$k_{\text{gr}}(t) = -\frac{\partial}{\partial t} \ln \langle \Psi_{c \text{ int}}^E(t) | \Psi_{c \text{ int}}^E(t) \rangle = \frac{1}{\hbar} \frac{\sum_k \Gamma_k c_{ck}^2 e^{-\Gamma_k t/\hbar}}{\sum_k c_{ck}^2 e^{-\Gamma_k t/\hbar}}. \quad (15)$$

The decay properties of the resonance states can be studied best when their excitation takes place in a time interval that is very short as compared to the lifetime τ_{λ} of the resonance states. In such a case, no perturbation of the decay process by the still continuing excitation process will take place.

For an isolated resonance state k , (15) passes into the standard expression $k_{\text{gr}}(t) \rightarrow k_k = \Gamma_k/\hbar$. In this case, the quantity k_k is constant in time and corresponds to the standard relation $\tau_k = \hbar/\Gamma_k$ with $\tau_k = 1/k_k$. It describes the idealized case with exponential decay law and a Breit-Wigner resonance in the cross section.

Equation (15) describes, however, the decay rate also in the regime of overlapping resonances [23]. The overlapping and mutual influence of resonance states is maximal at the avoided (and true) crossing points in the complex plane where two eigenvalues z_k and $z_{k'}$ of the effective Hamilton operator H_{eff} coalesce (or almost coalesce). Nevertheless, the decay rate is everywhere smooth as can be seen also directly from (15). This result coincides with the general statement according to which all observable quantities behave smoothly at singular points.

An interesting result is the saturation of the average decay rate k_{av} in the regime of strongly overlapping resonances. According to the bottle-neck picture of the transition state theory, it starts at a certain critical value of bound-continuum coupling [24, 25]. As has been shown in [26], this is caused by width bifurcation since the definition of an average lifetime of the resonance states is meaningful only for either the long-lived states or the short-lived ones. The widths Γ_k of the long-lived (trapped) states are almost the same for all the different states k , i.e. $\Gamma_{\text{av}} \approx \Gamma_k$ for all long-lived resonance states [5]. It follows

therefore $k_{\text{av}} \approx \Gamma_{\text{av}}/\hbar$ from (15). According to the average width Γ_{av} , the average lifetime of the long-lived states can be defined by $\tau_{\text{av}} = 1/k_{\text{av}}$. Then $\tau_{\text{av}} = \hbar/\Gamma_{\text{av}}$. That means, the basic relation between lifetimes and decay widths of resonance states holds not only for isolated resonance states but also for the narrow (trapped) resonance states, i.e. for Γ_{av} and τ_{av} .

VI. RESONANCE TRAPPING AND DYNAMICAL PHASE TRANSITIONS

Some years ago, the question has been studied [11] whether or not the resonance trapping phenomenon is related to some type of phase transition. The study is performed by using the toy model

$$H_{\text{eff}}^{\text{toy}} = H_0 + i\alpha VV^+ \quad (16)$$

where H^0 and VV^+ are Hermitian, V is the coupling vector of the system to the environment and the parameter α simulates the coupling strength between system and environment. The calculations are carried out for the one-channel case and with the assumption that (almost) all crossing (exceptional) points accumulate in one point [27]. The control parameter α is a real number. It has been found that resonance trapping may be understood, in this case, as a second-order phase transition. The calculations are performed for a linear chain consisting of a finite number N of states. The state in the center of the spectrum traps the other ones and becomes a collective state in a global sense: it contains components of almost all basic states of the system, also of those which are not overlapped by it. The normalized width Γ_0/N of this state can be considered as the order parameter: it increases linearly as a function of α , and the first derivative of Γ_0/N jumps at the critical value $\alpha = \alpha^{\text{cr}}$. The two phases of the system differ by the number of localized states. In the case considered, this number is N at $\alpha < \alpha^{\text{cr}}$, and $N - 1$ at $\alpha > \alpha^{\text{cr}}$.

Much more interesting is the realistic case with the Hamiltonian (6). In this case, trapping of resonance states occurs in the regime of overlapping resonances hierarchically, i.e. one by one [5]. The crossing points do *not* accumulate in one point, but are distributed over a certain range of the parameter: a *dynamical phase transition* takes place in a finite parameter range inside the regime of overlapping resonances. It can therefore be observed [5]. Also in this case, almost all resonance states are involved in the phase transition of the system and,

furthermore, the number N of localized states is reduced.

The dynamical phase transition taking place in the system at high level density causes finally a *dynamical stabilization* of the system: the system consisting of only the localized long-lived states beyond the phase transition is more stable than the system below the dynamical phase transition in spite of the stronger coupling between system and environment beyond the phase transition. The reason is the following: first the widths bifurcate at high level density and then the state with the shortest lifetime is ejected. The dynamical stabilization is a global effect to which all states contribute collectively by aligning one of the states (step by step) with a decay channel. In this manner, the sum of the decay widths of the states of the system is reduced [28], and the system is stabilized.

VII. DYNAMICAL STABILIZATION OF DIFFERENT QUANTUM SYSTEMS IN EXPERIMENTAL DATA

A. Phase lapses

More than 10 years ago, in experiments on Aharonov-Bohm rings containing a quantum dot in one arm, both the phase and the magnitude of the transmission amplitude $T = |T| e^{i\beta}$ of the dot are extracted [29]. The results obtained caused much discussion since they do not fit into the standard understanding of the transmission process. As a function of the plunger gate voltage V_g , a series of well-separated transmission peaks of rather similar width and height has been observed in many-electron dots and, according to expectations, the transmission phases $\beta(V_g)$ increase continuously by π across every resonance. In contrast to expectations, however, β always jumps sharply downwards by π in each valley between any two successive peaks. These jumps called phase lapses, were observed in a large succession of valleys for every many-electron dot studied. Only in few-electron dots, the expected so-called mesoscopic behavior is observed, i.e. the phases are sensitive to details of the dot configuration. The problem is considered theoretically in many papers over many years without solving it convincingly, e.g. [30].

In [31], the generic features of phase lapses in the inelastic cross section are studied by using the toy model (16) for the non-Hermitian Hamilton operator. According to the results of these calculations, the universal features observed in the phase lapses at high level

density, in contrast to the mesoscopic features at low level density, may be considered to be a hint at a dynamical phase transition. The transition occurs by controlling the system from low to high level density simulated in the calculations by means of α . In accordance to this picture, only the resonance states at low level density show individual spectroscopic features. At high level density, the observed resonances arise from trapped states. They show level repulsion, have vanishing spectroscopic relation to the open decay channels (i.e. small decay widths), and phase lapses appear. It follows further, that any theoretical study on the basis of conventional Hermitian quantum physics is unable to explain the experimental results convincingly. More accurate calculations on the basis of (7) for the non-Hermitian Hamiltonian H_{eff} are performed recently [32] and compared with the experimental data.

B. Spin swapping operation

A swapping gate in a two-spin system exchanges the degenerate states $|\uparrow, \downarrow\rangle$ and $|\downarrow, \uparrow\rangle$. Experimentally, this is achieved by turning on and off the spin-spin interaction b that splits the energy levels and induces an oscillation with a natural frequency ω . An interaction \hbar/τ_{SE} with an environment of neighboring spins degrades this oscillation within a decoherence time scale τ_ϕ . The experimental frequency ω is expected to be roughly proportional to b/\hbar and the decoherence time τ_ϕ proportional to τ_{SE} . In [13], experimental data are presented that show drastic deviations in both ω and τ_ϕ from this expectation. Beyond a critical interaction with the environment, the swapping freezes and the decoherence rate drops as $1/\tau_\phi \propto (b/\hbar)^2 \tau_{SE}$. That means, the relaxation decreases when the coupling to the environment increases. The transition between these two quantum dynamical phases occurs when $\omega \propto \sqrt{(b/\hbar)^2 - (k/\tau_{SE})^2}$ becomes imaginary (where k depends only on the anisotropy of the system-environment interaction, $0 \leq k \leq 1$). The experimental results are interpreted by the authors as an environmentally induced quantum dynamical phase transition occurring in the spin swapping operation [13].

Further theoretical studies within the Keldysh formalism showed that τ_ϕ is a non-trivial function of the system-environment interaction rate τ_{SE} , indeed: it is $1/\tau_\phi \propto 1/\tau_{SE}$ at low τ_{SE} (according to the Fermi golden rule) but $1/\tau_\phi \propto \tau_{SE}$ at large τ_{SE} . This theoretical result is in (qualitative) agreement with the experimental results. In [13], the dynamical phase transition in the spin swapping operation is related to the existence of an exceptional point.

The dynamical phase transition observed experimentally in the spin swapping operation and described theoretically within the Keldysh formalism shows qualitatively the same features as the dynamical phase transitions discussed in the present paper on the basis of the resonance trapping phenomenon (width bifurcation).

C. Loss induced optical transparency in complex optical potentials

The optical wave equation for complex PT symmetric potentials is formally equivalent to the quantum mechanical Schrödinger equation [33]. One expects therefore that PT symmetric optical lattices show a behavior which is qualitatively similar to that discussed for open quantum systems.

Experimental studies showed, indeed, a phase transition that leads to a loss induced optical transparency in specially designed non-Hermitian guiding potentials [20]: the output transmission first decreases, attains a minimum and then increases with increasing loss. The phase transition is related, in these papers, to PT symmetry breaking. In a following theoretical paper [34], the Floquet-Bloch modes are investigated in PT symmetric complex periodic potentials. As a result, the modes are skewed (nonorthogonal) and nonreciprocal. That means, they show the same features as modes of an open quantum system under the influence of exceptional points. A detailed discussion of this analogy is given in [12].

D. Dicke superradiance and subradiance in optics

The assumption that the probability of a given molecule to emit a photon may be considered to be independent of the states of the other molecules is justified only when the distance between the molecules is large. Generally, all the molecules are interacting with the common radiation field and the spontaneous emission takes place coherently. Dicke [35] was the first who considered the coherence in spontaneous radiation processes and, as a consequence, the formation of the so-called superradiant state. The collective coupling of the atoms via the radiation field leads also to a substantial radiative shift of the transition energy, the so-called collective Lamb shift. This effect is recently proven experimentally [8]. It will allow us to probe aspects of quantum electrodynamics in relatively low-energy experiments [9].

By using the simple non-Hermitian Hamilton operator (16), the formation of a superradi-

ant state can be controlled by the coupling strength α between system and environment [36]. At large α , the short-lived superradiant state is formed together with long-lived subradiant (trapped) states. The corresponding phase transition, called superradiance transition by the authors, is nothing but the dynamical phase transition discussed in section VI. In both cases, the same Hamiltonian with frozen internal degrees of freedom is used. In section VI the relation to exceptional points is considered.

Using an effective non-Hermitian Hamilton operator describing photon-mediated atomic dipolar interactions, calculations for the Dicke superradiance in atomic gases are performed recently [37]. The calculations show all the features characteristic of a dynamical phase transition. Only the notations used are different: for example 'disorder' used in [37] corresponds to 'individual spectroscopic properties of the states' in the many-body problem (sections II and III). Also in these calculations, the crossover from the uncorrelated to the correlated behavior appears at a critical value of a certain parameter. Interesting is the scaling behavior of the escape rates of photons propagating in a 3D atomic gas when traced by means of a certain parameter. For small parameter values, cooperative effects are negligible, and photons are emitted in spontaneous and incoherent processes. For larger values of this parameter, cooperative effects set in and the Dicke superradiance appears. The corresponding subradiance is considered in [38]. It extends the lifetime of the excitation to many times the natural lifetime of a single atom and is, therefore, doubtless interesting in quantum information science.

VIII. TIME IN AN OPEN QUANTUM SYSTEM

The results of sections IV to VII show that resonance states with very large widths, corresponding to very short lifetimes, do not exist. Due to the avoided crossings of the eigenvalue trajectories and the accompanying reduced phase rigidity $\rho < 1$, most resonance states are dynamically localized at high level density and have a finite non-vanishing lifetime. The following conclusions can be drawn.

- (i) Energy and time are determined by the eigenvalues of one and the same operator, namely of H_{eff} . The time operator is the non-Hermitian part of H_{eff} while the energy operator corresponds, as well known, to the Hermitian part of H_{eff} .
- (ii) Not only time varies continuously (as well known) but also energy does so, since most

states of an open quantum system are resonance states with a non-vanishing width.

(iii) Time is bounded from below in a similar manner as energy.

(iv) Discrete states correspond to $t \rightarrow \infty$. This relation is analog to the assumptions of standard quantum mechanics described by Hermitian operators.

The two operators $\text{Re}(H_{\text{eff}})$ and $\text{Im}(H_{\text{eff}})$ do not commute when the number C of channels is different from the number N of states, as can be seen easily from (16). Usually $C \ll N$. According to the points (i) to (iv), the main argument by Pauli [1] against the derivation of the time energy uncertainty relation does not exist when an open quantum system with the non-Hermitian Hamilton operator H_{eff} is considered.

The concept of time considered in the present paper is characteristic of an open quantum system in the same manner as energy. It has a physical meaning only for $t_b < t < \infty$ where t_b is the time at which width bifurcation creates the dynamic stabilization of the system. The time t defined in this manner is a measurable quantity. Note t is more sensitive to parameter variations than E (see [39] and [53] for numerical examples).

The dynamical stabilization of the system at high level density appears as a counter-intuitive process when time is considered as a parameter that may be varied continuously between $-\infty$ and $+\infty$ and is *not characteristic* of the system. In contrast to this, the finite value t_b relies on the fact that time is a value *characteristic* of the system. The dynamical stabilization occurs in consequence of the fact that the resonances will never really overlap. The only way to achieve this is to accumulate almost all coupling strength between system and environment onto one state (in the one channel case) while the remaining states become stabilized (long-lived) [40]. The short-lived state created in this manner, is aligned to the continuum of scattering states and does not belong to the set of localized states. Beyond t_b , the system differs from the original one: the number of states is reduced and their individual spectroscopic features are lost. Instead, cooperative effects are important (which may be disturbed only by coupling the system to another channel). Mathematically, the redistribution rests on the existence of exceptional points and the related phenomenon of avoided level crossings. It occurs under the influence of the environment into which the system is embedded.

Experimentally it is possible to form spatially remote discrete states on separate quantum point contacts and to allow an interaction between them via a common continuum [42]. The results show that the continuum supports an effective interaction between the two states

which is mediated by the continuum and is highly robust: the detector exhibits two Fano resonances due to the two different bound states. These studies, revealing a pronounced avoided level crossing, show clearly that the continuum affects the microscopic structure of bound states in the two quantum point contacts.

In the examples discussed in section VII, the time t_b appears in a natural manner. It causes *measurable* effects. For example, the transmission through a small quantum system is enhanced in the parameter range in which the redistribution in the system takes place [41]. Interesting are the very stable whispering gallery modes in a small quantum system. They are partly aligned to the scattering wavefunctions and their lifetimes are shorter than those of the other states. They cause an enhancement of the transmission, and the system becomes almost transparent ($\rho \rightarrow 0$) under these conditions.

The time t_b determines also the brachistochrone problem [5] which consists of finding the minimal time for the transition from a given initial state to a given final state of the considered system. At high level density, the individual resonance states can no longer be identified. Here $\rho < 1$, and the wavefunctions of some states of the system are partly aligned to the scattering wavefunctions such that the time for the transition from a given initial state to a given final state may be radically shortened. However, the time for traveling through the system does never vanish. It is bounded from below since it can not be smaller than the time corresponding to the transparency of the system [5].

The non-adiabatic processes found recently when cycling exceptional points [43], are surely related to the finite time t_b below which time loses its meaning in the system considered. The cycling crosses regions with fundamental different time concepts.

IX. SUMMARY

In the present paper, the meaning of time in an open quantum system is considered. System as well as environment are quantum mechanical objects with the consequence that the Hamilton operator H_{eff} of the system is non-Hermitian. The real parts of the eigenvalues of H_{eff} provide the energies of the states while the imaginary parts of them are related to their lifetimes. Time t in the open quantum system is defined by the lifetimes of the decaying states.

In an open quantum system described by a non-Hermitian operator, the main objection

[1] to the derivation of the time energy uncertainty relation does not occur. The time operator appears in a natural manner together with the energy operator. Energy *and* time are bounded from below and, furthermore, vary continuously as a function of a parameter, as a rule.

The open quantum system is reversible at low level density where the levels are far from one another and the system can be described, to a good approximation, by a Hermitian operator. At high level density, however, the levels avoid crossing and irreversible processes determine the redistribution processes taking place in the system. These irreversible processes are caused by nonlinearities in the Schrödinger equation of the open system in the vicinity of avoided level crossings (and exceptional points, respectively) due to the coupling of the states via the environment. A dynamical stabilization of the system occurs, at the time t_b , under the influence of the environment. t_b is the lowest value of t .

Further experimental as well as theoretical studies are necessary. Above all, the time energy uncertainty relation has to be derived in a mathematical convincing manner for the general case with discrete and narrow resonance states of a many-particle system. Experimentally, the influence of a third state onto the mixing of two states in the neighborhood of an avoided level crossing should be studied. As discussed in appendix E, this is the basic process of the dynamical stabilization taking place in the system at high level density.

Appendix

Appendix A: The eigenvalues of a non-Hermitian 2×2 operator

Let us consider the Hamiltonian

$$H(\omega) = \begin{pmatrix} \epsilon_1 & \omega \\ \omega & \epsilon_2 \end{pmatrix} \quad (\text{A1})$$

with the energies ϵ_i ($i = 1, 2$) of the two states and the interaction ω between them. The ϵ_i are assumed to contain the corrections due to the coupling of the state i to the environment. The eigenvalues are

$$\epsilon_{1,2} = \frac{\epsilon_1 + \epsilon_2}{2} \pm Z ; \quad Z = \frac{1}{2} \sqrt{(\epsilon_1 - \epsilon_2)^2 + 4\omega^2} . \quad (\text{A2})$$

The levels repel each other in energy according to the value $\text{Re}(Z)$ while the widths bifurcate corresponding to $\text{Im}(Z)$. The two eigenvalue trajectories cross when $Z = 0$, i.e. when $(\epsilon_1 -$

$\epsilon_2)/2\omega = \pm i$. At the crossing points, called mostly *exceptional points*, the two eigenvalues coalesce, $\epsilon_1 = \epsilon_2 \equiv \epsilon_0$. In the vicinity of the crossing points, the dependence of the eigenvalue trajectories on the parameter is more complicated than far from them: the two levels approach each other in energy and the widths become equal so that $\text{Re}(\epsilon_1) \leftrightarrow \text{Re}(\epsilon_2)$ and $\text{Im}(\epsilon_1) \leftrightarrow \text{Im}(\epsilon_2)$ at the crossing point.

When H is a Hermitian operator, the unperturbed energies ϵ_i of the states and the interaction ω between them are real. According to (A2), the two (real) eigenvalue trajectories $\epsilon_i(\alpha) = e_i(\alpha)$ cannot cross (for $\omega \neq 0$) when traced as a function of a certain parameter α . Instead, they avoid crossing. The fictive crossing point is called usually *diaboloic point*. The topological structure of this point is characterized by the Berry phase [44] which is studied theoretically and experimentally in many papers.

The situation is another one when H is a non-Hermitian operator. In such a case, the unperturbed energies ϵ_i and also the interaction ω are complex, usually. The states can decay, in general, and the two eigenvalues (A2) can be written as

$$\epsilon_{1,2} = e_{1,2} - \frac{i}{2} \gamma_{1,2} \quad (\text{with } \gamma_{1,2} \geq 0) . \quad (\text{A3})$$

The widths γ_i are proportional to the inverse lifetimes τ_i^{-1} of the states, $i = 1, 2$. The two eigenvalue trajectories $\epsilon_i(\alpha)$ may cross according to (A2), and the crossing point is an exceptional point in agreement with the definition given in [45]. The topological phase of the exceptional point is twice the Berry phase [5]. This theoretical result is proven experimentally by means of a microwave cavity [46]. According to (A2), $\text{Re}(Z)$ causes repulsion of the levels in energy. It is the dominant part when the interaction $|\omega|$ of the states is small. The value $\text{Im}(Z)$ is dominant when $|\omega|$ is large. It is related, according to (A2), to a bifurcation of the widths of the levels.

The formal equivalence of the optical wave equation in PT symmetric optical lattices to the quantum mechanical Schrödinger equation allows us to study the properties of quantum systems the states of which can not only decay due to their coupling to the environment according to (A3), but may also be formed out of the environment [20]. In optics, these two possibilities are called *loss* and *gain*. In PT symmetric optical lattices, the eigenvalues are [12]

$$\epsilon_{1,2} = e_{1,2} \pm \frac{i}{2} \gamma_{1,2} \quad (\text{with } \gamma_{1,2} \geq 0 \text{ and } e_1 = e_2) \quad (\text{A4})$$

in difference to (A3). Due to PT symmetry, all eigenvalues $\varepsilon_i = e_i$ may be real (corresponding to $\gamma_i = 0$) when $\text{Re}(Z) \gg \text{Im}(Z)$, i.e. at low coupling of the states to the continuum. However, the PT symmetry is broken when $\text{Im}(Z) \gg \text{Re}(Z)$ and $\gamma_{1,2} \neq 0$. The difference between the two models with the eigenvalues (A3) and (A4) will allow us to receive interesting information on quantum systems by studying not only open quantum systems (which exist in nature) but also PT symmetric systems (which are formally equivalent to them).

Appendix B: The eigenfunctions of a non-Hermitian 2×2 operator

The eigenfunctions of the non-Hermitian Hamilton operator H , equation (A1), are biorthogonal,

$$\langle \phi_k^* | \phi_l \rangle = \delta_{k,l} . \quad (\text{B1})$$

From these equations follows

$$\langle \phi_k | \phi_k \rangle \equiv A_k \geq 1 \quad (\text{B2})$$

$$\langle \phi_k | \phi_{l \neq k} \rangle = -\langle \phi_{l \neq k} | \phi_k \rangle \equiv B_k^l ; \quad |B_k^l| \geq 0 . \quad (\text{B3})$$

At the crossing point $A_k^{(\text{cr})} \rightarrow \infty$, $|B_k^l|^{(\text{cr})} \rightarrow \infty$, for details see [5].

The relation between the eigenfunctions ϕ_1 and ϕ_2 of the operator (A1) at the crossing point is

$$\phi_1^{\text{cr}} \rightarrow \pm i \phi_2^{\text{cr}} ; \quad \phi_2^{\text{cr}} \rightarrow \mp i \phi_1^{\text{cr}} \quad (\text{B4})$$

according to analytical as well as numerical studies [5]. That means, the state ϕ_1 jumps, at the exceptional point, via the chiral state $\phi_1 \pm i\phi_2$ to the state $\pm i\phi_2$.

The two eigenfunctions are linearly dependent of one another at the crossing point such that the number of eigenfunctions of H seems to be reduced at this point. Theoretical studies [47] have shown however that *associated vectors* ϕ_i^{cra} defined by the Jordan relations, appear at the crossing points. The corresponding equations are

$$(H - \varepsilon_0) \phi_{1,2}^{\text{cr}} = 0 ; \quad (H - \varepsilon_0) \phi_{1,2}^{\text{cra}} = \phi_{1,2}^{\text{cr}} . \quad (\text{B5})$$

The existence of two states in the very neighborhood of the exceptional point has been seen in a numerical calculation for the elastic scattering of a proton on a light nucleus [48]: the

elastic scattering phase shifts jump always by 2π (and not by π as for a single resonance state).

In an experimental study on a microwave cavity [46], the topological structure of the exceptional point and its surrounding is studied by encircling it and tracing the relative amplitudes of the wavefunctions (field distributions inside the cavity). As a result, the wavefunctions including their phases are restored after four surroundings. The authors [46] interpreted the experimental data by two theoretical assumptions: (i) the two wavefunctions coalesce into one at the exceptional point, $\phi_1^{\text{cr}} \leftrightarrow \phi_2^{\text{cr}}$, and (ii) only one of the wavefunctions picks up a phase of π (a sign change) when encircling the critical point. The experimental result can be explained, however, without any additional assumptions by using the relations (B4):

1. cycle: $\varepsilon_{1,2} \rightarrow \varepsilon_{2,1}; \quad \phi_{1,2} \rightarrow \pm i \phi_{2,1};$
2. cycle: $\varepsilon_{2,1} \rightarrow \varepsilon_{1,2}; \quad \pm i \phi_{2,1} \rightarrow -\phi_{1,2}$
3. cycle: $\varepsilon_{1,2} \rightarrow \varepsilon_{2,1}; \quad -\phi_{1,2} \rightarrow \mp i \phi_{2,1};$
4. cycle: $\varepsilon_{2,1} \rightarrow \varepsilon_{1,2}; \quad \mp i \phi_{2,1} \rightarrow \phi_{1,2}.$

The eigenvalues are restored after two surroundings and the eigenfunctions are restored after four surroundings, in full agreement with the experimental result. In any case, $|\phi_1^{\text{cr}}| = |\phi_2^{\text{cr}}|$ at the crossing point. The topological phase is twice the Berry phase, in accordance with the enlarged function space in open quantum systems.

Furthermore, the phases of the wavefunctions jump by $\pi/4$ at the crossing point (when traced as a function of a parameter) due to the biorthogonality (B1) of the eigenfunctions of the non-Hermitian Hamiltonian H , see also (B2) and (B3). This result has been proven in many numerical studies, see [5].

Appendix C: The phase rigidity r_λ of the eigenfunctions of a non-Hermitian 2×2 operator

Let us now consider the consequences of the biorthogonality relations (B1) and (B2) for the two borderline cases characteristic of neighboring resonance states.

- (i) The two levels are distant from one another. Then the eigenfunctions are (almost) orthogonal, $\langle \phi_k^* | \phi_k \rangle \approx \langle \phi_k | \phi_k \rangle = A_k \approx 1$.
- (ii) The two levels cross. Then the two eigenfunctions are linearly dependent according to (B4) and $\langle \phi_k | \phi_k \rangle = A_k \rightarrow \infty$.

These two relations show that the phases of the two eigenfunctions relative to one another

change when the crossing point is approached. This can be expressed quantitatively by defining the *phase rigidity* r_k of the eigenfunctions ϕ_k ,

$$r_k \equiv \frac{\langle \phi_k^* | \phi_k \rangle}{\langle \phi_k | \phi_k \rangle} = A_k^{-1}. \quad (\text{C1})$$

It holds $1 \geq r_k \geq 0$. The non-rigidity r_k of the phases of the eigenfunctions of H follows also from the fact that $\langle \phi_k^* | \phi_k \rangle$ is a complex number (in difference to the norm $\langle \phi_k | \phi_k \rangle$ which is a real number) such that the normalization condition (B1) can be fulfilled only by the additional postulation $\text{Im}\langle \phi_k^* | \phi_k \rangle = 0$ (what corresponds to a rotation).

Note, discrete states never cross but always avoid crossing, see appendix A. The eigenfunctions are real and normalized as $\langle \phi_k | \phi_k \rangle = 1$ (see appendix A). Accordingly, $r_k = 1$ for discrete states also in the region of an avoided level crossing. The corresponding crossing (exceptional) point can be found by analytical continuation into the continuum [18].

The variation of r_k in approaching the crossing point of two eigenvalue trajectories of resonance states is proven experimentally by means of a study on a microwave cavity [49]. As a result, the phase difference between two modes is π at large distance and decreases to $\pi/2$ at the crossing point. The authors of [49] interpret the experimental data by assuming (i) that the singular point is a chiral state (in spite of the phase jump occurring at the crossing point, when traced as a function of a certain parameter, see (B4)), (ii) that the number of states is reduced from 2 to 1 at the crossing point (in spite of the existence of the associate vector (B5)) and (iii) that a single point in the continuum can be identified (although it is of measure zero). The authors are unable to explain the large parameter range in which the phase difference decreases in approaching the crossing point.

Considering the phase rigidity r_k in the regime of the two overlapping resonance states, no additional assumptions are required for the explanation of the experimental results given in [49], since the phase rigidity (being a quantitative measure for the degree of resonance overlapping) varies smoothly in a comparably large parameter range. It can therefore be concluded that the experimental results [49] prove the statement that the phases of the eigenfunctions of the non-Hermitian Hamilton operator H , equation (A1), are not rigid in approaching the crossing point.

Appendix D: Nonlinear source term in the Schrödinger equation in the neighborhood of an exceptional point

According to (A1), the Schrödinger equation with the unperturbed operator $H_0 \equiv H(\omega = 0)$ and a source term arising from the interaction ω with another state reads [50]

$$\begin{aligned} (H_0 - \epsilon_n) |\phi_n\rangle &= - \begin{pmatrix} 0 & \omega \\ \omega & 0 \end{pmatrix} |\phi_n\rangle \equiv W |\phi_n\rangle \\ &= \sum_{k=1,2} \langle \phi_k | W | \phi_n \rangle \{ A_k |\phi_k\rangle + \sum_{l \neq k} B_k^l |\phi_l\rangle \}. \end{aligned} \quad (\text{D1})$$

Here $\langle \phi_k | \phi_k \rangle \equiv A_k \geq 1$ according to (B2) and $\langle \phi_k | \phi_{l \neq k} \rangle = -\langle \phi_{l \neq k} | \phi_k \rangle \equiv B_k^l$, $|B_k^l| \geq 0$ according to (B3). The A_k and B_k^l characterize the degree of resonance overlapping. In the regime of overlapping resonances, $1 > A_k > 0$, $|B_k^l| > 0$, and equation (D1) is nonlinear. The most important part of the nonlinear contributions is contained in

$$(H_0 - \epsilon_n) |\phi_n\rangle = \langle \phi_n | W | \phi_n \rangle |\phi_n\rangle^2 |\phi_n\rangle \quad (\text{D2})$$

which is a nonlinear Schrödinger equation. According to (D1), the nonlinear Schrödinger equation (D2) passes smoothly into the standard linear Schrödinger equation when $A_k \rightarrow 1$ and $B_k^l \rightarrow 0$.

Appendix E: Time reversal symmetry breaking in the neighborhood of an exceptional point

Exceptional points that are well separated from the influence of external sources (including the influence caused by other resonance states), are highly symmetric in approaching them. That means, the two states pass one into the other one according to (B4) with an exchange of their wavefunctions, $\phi_1^{\text{cr}} \rightarrow \pm i \phi_2^{\text{cr}}$ and $\phi_2^{\text{cr}} \rightarrow \mp i \phi_1^{\text{cr}}$. At a certain finite distance from the exceptional point, there are again two states with the wavefunctions $|\phi_1|$ and $|\phi_2|$, respectively.

This symmetry may be distorted under the influence of an external magnetic field as has been shown experimentally on a microwave cavity [51]. The magnetic field causes time reversal symmetry breaking.

The symmetry may be disturbed also by the influence of another resonance state in the neighborhood due to the finite parameter range around the exceptional point in which the

wavefunctions of the two states are mixed with each other [50]. When the interaction of the third state is symmetric relative to the two crossing ones, the third state will appear as an *observer* and time reversal symmetry is not broken. Numerical examples of such a situation are shown in the transmission through a quantum dot [52] and also in the generic case studied in [53]. When the interaction of the third state with the two crossing ones is, however, not symmetrically, time reversal symmetry may be broken and may cause irreversible processes due to the nonlinear terms in the Schrödinger equation as discussed in appendix D.

It would be highly interesting to study experimentally time reversal symmetry breaking in the case of an exceptional point disturbed non-symmetrically by a third state in the neighborhood.

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